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EFFICIENT RELIABILITY AND UNCERTAINTY ASSESSMENT ON LIFELINE NETWORKS USING THE SURVIVAL SIGNATURE

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Abstract. *Lifeline networks, such as water distribution and transportation networks, are the backbone of our societies, and the study of their reliability of them is required. In this paper, a survival signature-based reliability analysis method is proposed to analyse the complex networks. It allows to consider all the characters of the network instead of just analysing the most critical path. What is more, the survival signature separates the system structure from its failure distributions, and it only needs to be calculated once, which makes it efficient to analyse complex networks. However, due to lack of data, there often exists imprecision within the network failure time distribution parameters and hence the survival signature. An efficient algorithm which bases on the reduced ordered binary decision diagrams (BDD) data structure for the computation of survival signatures is presented. Numerical example shows the applicability of the approaches.*

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1 INTRODUCTION

Nowadays reliability engineering is used in a wide range of applications on complex lifeline networks, which are a series of components interconnected by communication paths. The analysis of these networks becomes more and more important as they are the backbone of our societies. Examples include the Internet, social networks of individuals or businesses, transportation network, power plant system, aircraft and space flights, metabolic networks, and many others. Since the breakdown of lifeline networks might have catastrophic effects, it is essential to assess the reliability and availability of such networks.

System signatures [1] have been recognized as an important tool to quantify the reliability of systems, however, the use of the system signature is associated with the assumption that all components in the system are of the same type. Survival signature which does not rely any more on the restriction to one component type has been proposed by Coolen and Coolen-Maturi in [2]. Recent developments have opened up a pathway to perform a survival analysis using the concept of survival signature even for complex lifeline networks. Aslett [3] developed a Reliability Theory R package which was used to calculate the survival signature. Feng et al. [4] considered imprecise system reliability and component importance measures. Patelli and Feng [5] proposed efficient simulation approaches based on survival signature for reliability analysis of large systems. Coolen and Coolen-Maturi [6] linked the (imprecise) probabilistic structure function to the survival signature. An algorithm for exact computation of system and survival signatures is proposed by Reed [7].

Most existing models assume that there are precise parameter values available, so the quantification of uncertainty is mostly done by the use of precise probabilities [8]. However, due to lack of perfect knowledge, imprecision often exists within component failure times or their distribution parameters. Hence, the reliability analysis for the lifeline network is affected by the imprecision and uncertainty.

Augustin et al. gave a detailed introduction of imprecise probability in [9]. In order to deal with the uncertainty, Beer et al. [10] introduced fuzzy set theory in engineering analyses. An integrated framework to deal with scarce data, aleatory and epistemic uncertainties is presented by Patelli et al. [11], and OpenCossan is an efficient tool to perform uncertainty management of large finite element models [12]. Also, the use of probability box in risk analysis offers many significant advantages over a traditional probabilistic approaches because it provides convenient and comprehensive ways to handle several of the most practical serious problems face by analysts [13].

In this paper, survival signature is used perform reliability and uncertainty assessment on complex networks. The remainder of the paper is organized as follows. Section 2 gives a brief overview of survival signature-based reliability analysis on lifeline networks with imprecision. Then, an efficient algorithm which bases on the reduced ordered binary decision diagrams (BDD) data structure for calculating the survival signature is proposed in Section 3. In Section 4, a numerical example is analysed to show the performance and applicability of the proposed methods. Finally, the paper is concluded in Section 5 with some discussions.

2 RELIABILITY ANALYSIS ON LIFELINE NETWORKS WITH IMPRECISION

2.1 Reliability Assessment on Lifeline Networks

Lifeline network may be represented with m components which belong to $K \geq 2$ component types, with m_k components of type $k \in \{1, 2, ..., K\}$ and $\sum_{k=1}^{K} m_k = m$. Assume that the random failure times of components of the same type are exchangeable, while full independence is assumed for components belong to different types (iid), the survival signature which can be denoted by $\Phi(l_1, l_2, ..., l_K)$, with $l_k = 0, 1, ..., m_k$ for $k = 1, 2, ..., K$. It defines the probability that the system functions given that l_k of its m_k components of type k work, for each $k \in \{1, 2, ..., K\}$. There are $\binom{m_k}{l_k}$ state vectors $\underline{x}^k$ with $\sum_{i=1}^{m_k} x_i^k = l_k$ ($k = 1, 2, ..., K$), where $\underline{x}^k = (x_1^k, x_2^k, ..., x_{m_k}^k)$. Let $S_{l_1,l_2,...,l_K}$ denote the set of all state vectors for the whole system, and it can be known that all the state vectors $\underline{x}^k \in S_{l_k}^k$ are equally likely to occur. Therefore, the survival signature can be expressed as:

$$\Phi(l_1, ..., l_K) = [\prod_{k=1}^{K} \binom{m_k}{l_k}^{-1}] \times \sum_{\underline{x} \in S_{l_1,...,l_K}} \phi(\underline{x}) \tag{1}$$

where $\phi = \phi(\underline{x}) : \{0,1\}^m \to \{0,1\}$ is the system structure function, i.e., the system status based on all possible state vectors $\underline{x}$. ϕ is 1 if the system functions for state vector $\underline{x}$ and 0 if not.

Let $C_k(t) \in \{0, 1, ..., m_k\}$ denote the number of k components working at time t, the survival function of the system with K types of components becomes:

$$P(T_s > t) = \sum_{l_1=0}^{m_1} ... \sum_{l_K=0}^{m_K} \Phi(l_1, ..., l_K) P(\bigcap_{k=1}^{K} \{C_k(t) = l_k\}) \tag{2}$$

If the components of type k have a known cumulative distribution function $F_k(t)$, let make the assumptions of independence of failure times for components of different types and of iid given a distribution function for components of the same type, then:

$$P(\bigcap_{k=1}^{K} \{C_k(t) = l_k\}) = \prod_{k=1}^{K} P(C_k(t) = l_k) = \prod_{k=1}^{K} \binom{m_k}{l_k} [F_k(t)]^{m_k - l_k} [1 - F_k(t)]^{l_k} \tag{3}$$

Equation 2 shows that the structure of the system is separated from the its components failure times, which is the typical advantage of the survival signature. The survival signature is a summary of structure functions and only needs to be calculated once for the same system. As a result, it is an efficient method to perform system reliability analysis on complex systems with multiple component types.

2.2 Uncertainty Assessment on Lifeline Networks

Uncertainty is an unavoidable component affecting the behaviour of systems and more so with respect to their limits of operation. In spite of how much effort is dedicated into improving the understanding of systems, components and processes through the collection of representative data, the appropriate characterization, representation, propagation and interpretation of uncertainty will remain a fundamental element of the reliability analysis of any complex systems [14]. If only few data points are available it might be difficult to identify the parameters of the components precisely. However, it is essential to take the uncertainty into account when analyse the lifeline network reliability.

Considering the imprecision in the component parameters will lead to bounds of the Lifeline network survival function. The lower bound of the survival function can be got through Equation 4.

$$\underline{P}(T_S > t) = \sum_{l_1=0}^{m_1} \dots \sum_{l_K=0}^{m_K} \Phi(l_1, ..., l_K) \prod_{k=1}^{K} \overline{D}(C_k(t) = l_k) \tag{4}$$
$$= \sum_{l_1=0}^{m_1} \dots \sum_{l_K=0}^{m_K} \Phi(l_1, ..., l_K) \prod_{k=1}^{K} (\overline{P}(C_k(t) \leq l_k) - \overline{P}(C_k(t) \leq l_k - 1))$$

While the corresponding upper bound of the survival function can be calculated as:

$$\overline{P}(T_S > t) = \sum_{l_1=0}^{m_1} \dots \sum_{l_k=0}^{m_K} \Phi(l_1, ..., l_K) \prod_{k=1}^{K} \underline{D}(C_k(t) = l_k) \tag{5}$$
$$= \sum_{l_1=0}^{m_1} \dots \sum_{l_k=0}^{m_K} \Phi(l_1, ..., l_K) \prod_{k=1}^{K} (\underline{P}(C_k(t) \leq l_k) - \underline{P}(C_k(t) \leq l_k - 1))$$

3 PROPOSED ALGORITHM FOR CALCULATING SURVIVAL SIGNATURE OF LIFELINE NETWORKS

The state vector count or survival signature values for a system, such as a lifeline network, can be represented by a multidimensional array. The values stored at index $(l_1, ..., l_K)$ of the array stores the value corresponding to $l_1, ..., l_K$ components of types 1 to K surviving. However, computing these arrays using enumerative methods becomes quickly infeasible since the number of state vectors to consider is equal to 2^m and therefore the computational complexity grows exponentially with the number of components in the network. An efficient algorithm for computing the multidimensional array representation of the survival signature for a system, based on the use of the reduced ordered binary decision diagrams (BDD) data structure, is proposed.

A BDD [15] is a data structure in the form of a rooted directed acyclic graph which can be used to compactly represent and efficiently manipulate a Boolean function. They are based upon Shannon decomposition theory [16]. The Shannon decomposition of a Boolean function f on Boolean variable x_i is defined as $f = x_i \wedge f_{x_i=1} + \overline{x_i} \wedge f_{x_i=0}$ where $f_{x_i=v}$ is f evaluated with $x_i = v$. Each BDD contains two terminal nodes that represent the Boolean constant values 1 and 0, whilst each non-terminal node represents a subfunction g, is labelled with a Boolean variable v and has two outgoing edges. By applying a total ordering on the m Boolean variables for function f by mapping them to the integers $x_0, , x_{m-1}$, and applying the Shannon decomposition recursively to f, it can be represented as a binary tree with $m + 1$ levels. Note that the chosen ordering can have a significant influence on the size of the BDD [17]. Each intermediate node, referred to as an if-then-else (ite) node, at level $l \in \{0, , m - 1\}$ (where the root node is at level 0 and the nodes at level $m - 1$ are adjacent to the terminal nodes) represents a Boolean function g on variables $x_l, x_{l+1}, ..., x_{m-1}$. It is labelled with variable x_l and has two out edges called 1-edge and 0-edge linking to nodes labelled with variables higher in the ordering. 1-edge corresponds to $x_l = 1$ and links to the node representing $g_{x_l=1}$, whist 0-edge corresponds to $x_l = 0$ and links to the node representing $g_{x_l=0}$. In addition, the following two reduction rules are applied. Firstly, the isomorphic subgraphs are merged; and secondly, any node whose two children are isomorphic is eliminated.

Complement edges [18] are an extension to standard BDDs that reduce memory size and the computation time. A complement edge is an ordinary edge that is marked to indicate that the connected child node (at a higher level) has to be interpreted as the complement of its Boolean function. The use of complement edges is limited to the 0-edges to ensure canonicity.

The BDD representing the system structure function for a network can be computed in various ways, e.g. from its cut-sets or network decomposition based methods [19]. In order to show the implementation of the approach, a simple network with 4 nodes and 4 edges is considered and shown in Figure 1.

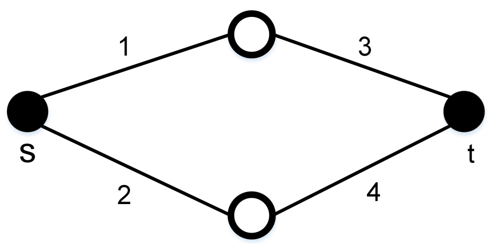

Figure 1: A simple network with 4 nodes and 4 edges.

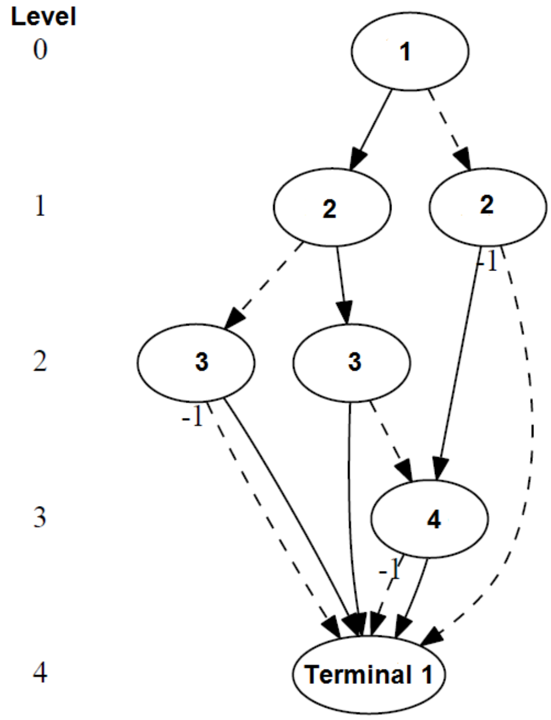

Figure 2: BDD for the simple network from Figure 1.

The corresponding BDD representing the structure function of this network, where the dashed edges represent 0-edges (marked with -1 if complemented) and solid edges represent 1-edges, is shown in Figure 2. The survival signature from a BDD representation of the system structure function for a network can then be calculated through the iterative algorithm described by Figure 3.

The number of operations performed during the execution of the algorithm grows approximately linearly with the number of nodes in the BDD. In general, the BDD representation of the structure function for a network has far fewer nodes than 2^m nodes. It is therefore far more computationally efficient than using enumerative algorithms.

4 NUMERICAL EXAMPLE

Figure 4 shows a lifeline network of 17 nodes and 32 edges. The source is the node s and the sink is the node t. All the nodes are assumed to be perfectly reliable in the network.

Three cases are considered. The first case is used to compare results between the former improved recursive decomposition method and the presented survival signature-based method. And the proposed approach is extended to analyse complex network with multiple component types in the second case. For the last case, imprecision is taken into consideration.

4.1 Network with Single Type of Components

Reliability analysis on the network show in Figure 4 considering only one type of components, which has been studied by Liu and Li [20]. In this Case, there has an assumption that the edges of the network are independent and identical distributed. All edges are undirected edges (which means all edges are connected by nodes), let all edges reliability be 0.9 (Case I), 0.8 (Case II), 0.2 (Case III) and 0.1 (Case IV).

The network reliability calculated by the improved recursive decomposition algorithm in [20] is 0.999930 (Case I), 0.996522 (Case II), 0.017194 (Case III) and 0.000777 (Case IV), respectively.

By using the efficient algorithm which proposed in this paper, the survival signature of the complex network can be calculated in 28.07 seconds, and the results can be seen in Table 1.

l	$\Phi(l)$	l	$\Phi(l)$	l	$\Phi(l)$	l	$\Phi(l)$	l	$\Phi(l)$	l	$\Phi(l)$
0	0	1	0	2	0	3	0	4	0.00014	5	0.00081
6	0.00285	7	0.0077	8	0.01765	9	0.03597	10	0.06683	11	0.00014
12	0.18409	13	0.27635	14	0.38916	15	0.51445	16	0.63944	17	0.75075
18	0.18409	19	0.90414	20	0.94679	21	0.97271	22	0.98719	23	0.99458
24	0.99799	25	0.99938	26	0.99985	27	0.99998	28	1	29	1
30	1	31	1	32	1						

Table 1: Survival signature of the network in Figure 4.

In all four Cases, the network reliabilities calculated through the survival signature-based reliability method given by Equation 2 are identical to those calculated using the method from Liu and Li [20]. However, the survival signature-based method only needs to calculate the survival signature of the network once and store the results, so it is efficient to calculate the network reliability for more cases. Furthermore, the proposed method is powerful at dealing with the complex networks with multiple component types and components with time varying distributions.

4.2 Network with Multiple Types of Components

Assume that the edges within the network are belonging to three types instead of one single type. To be specific, edges 1, 2, 3, 4, 5, 28, 29, 30, 31 and 32 are in type one with reliability is 0.9; edges 6, 7, 8, 9, 15, 16, 17, 18, 24, 25, 26 and 27 are in type two with reliability is 0.8; edges 10, 11, 12, 13, 14, 19, 20, 21, 22 and 23 are in type three with reliability is 0.2.

In order to estimate the network reliability, the survival signature of this network can be calculated by the proposed algorithm in 23.78 seconds, and then the reliability of the network is 0.3746931 by using Equation 2.

It can be seen from the above examples that the network reliability is time independent, because we assume the edge reliability values are stable as time goes. In the real engineering world, however, the failure times of edges are according to different distribution types (i.e., Exponential, Weibull, Normal and Lognormal distribution) sometimes. All of these distribution are time dependent, and will lead to the network reliability values are time varying.

Now let assume the failure times of edges type one are according to Exponential distribution with parameter $\lambda = 0.12$. Similarly, type two $\sim$ Weibull(0.4,4.2) and type three $\sim$ Normal(0.7,0.02).

The survival signature remains the same as the network does not change its configuration. The survival function of the network is shown in Figure 5. It can be seen that the survival function is time varying, thus, it is easy to know the network reliability at each time.

4.3 Imprecise Network Reliability

Due to lack of data or limited knowledge, there are not always precise data for edges failure time distributions. For instance, Table 2 shows the failure types and imprecise distribution parameters of edges in the network.

Edge Type	Distribution Type	Imprecise Parameters λ or (α, β)
1	Exponential	[0.08, 0.18]
2	Weibull	([0.3, 0.6], [3.8, 4.6])
3	Normal	([0.5, 0.8], [0.01, 0.03])

Table 2: Failure types and imprecise distribution parameters of edges in the network of Figure 4.

According to Equations 4 and 5, the lower and upper bounds of survival function of the network can be estimated by means of a double loop approach. The double loop sampling involves two layers of sampling: the outer loop, called the parameter loop, samples values from the set of distribution parameters; while the inner loop computes the survival function stating for the network knowing the precise probability distribution functions. Figure 5 shows the interval of the survival function. It can conclude that imprecision either within the components failure time distribution parameters can lead to survival function intervals of the complex network.

5 CONCLUSIONS

Survival signature opens a new pathway for analysing complex network with multiple component types, and it just needs to be calculated once for a specific network, which represents a significant computational advantage. An efficient algorithm has been proposed for calculating the survival signature of large and complex networks, and then used for reliability and uncertainty assessment on lifeline networks based on the survival signature.

The proposed approach allows to include imprecision and vagueness both within the components failure time distribution parameters and lifeline network configuration. Either analytical approaches or simulation methods can be applied to deal with the uncertainty efficiently. The case study presented in this paper indicates that the proposed approaches can be used to evaluate the reliability and uncertainty of complex networks efficiently.

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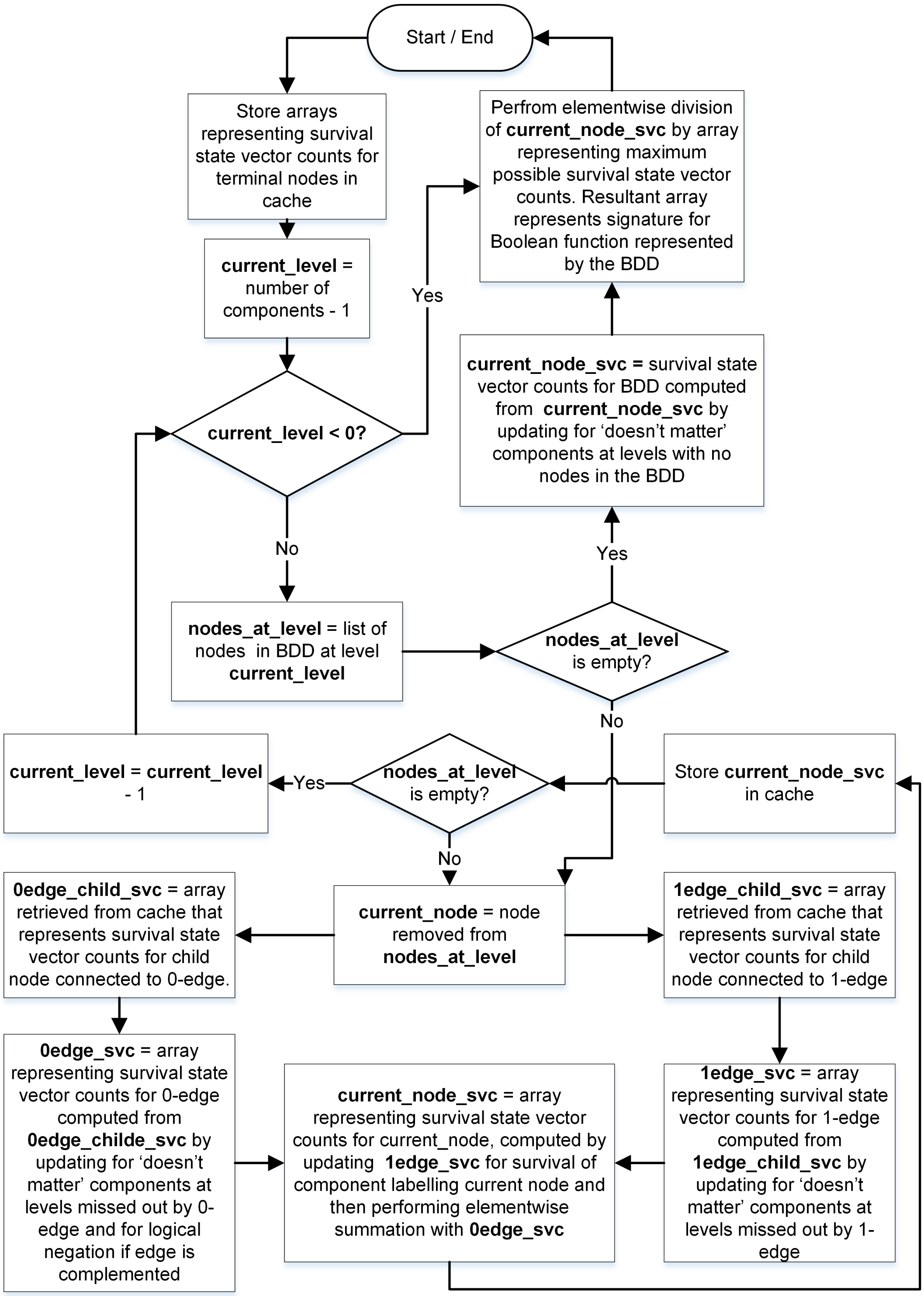

Figure 3: Algorithm for computing signature from the BDD representation of a system structure function.

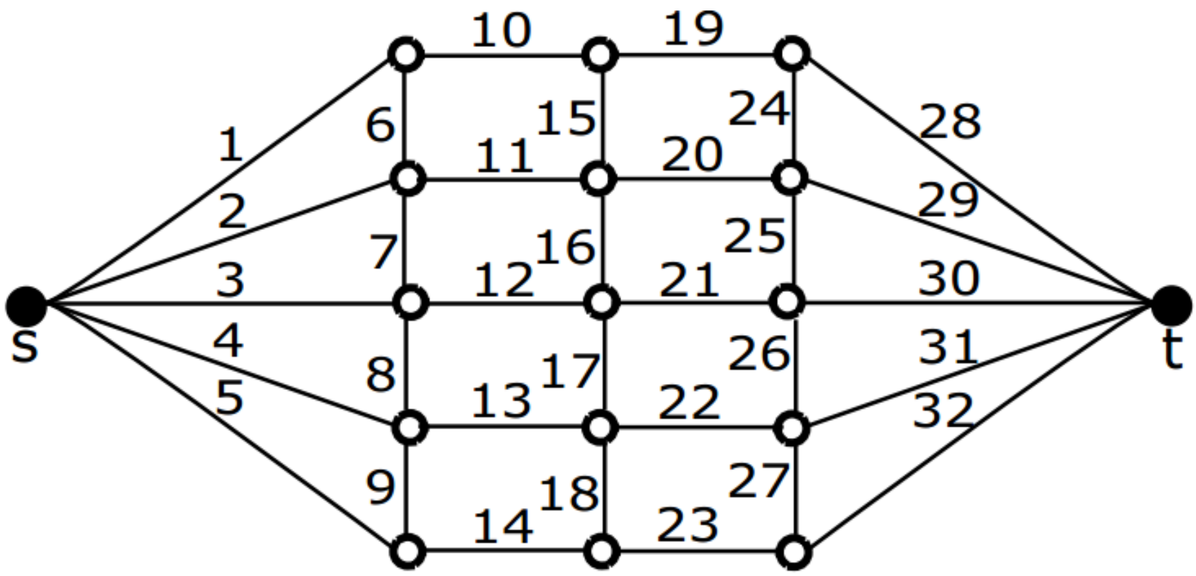

Figure 4: A lifeline network with 17 nodes and 32 edges [20].

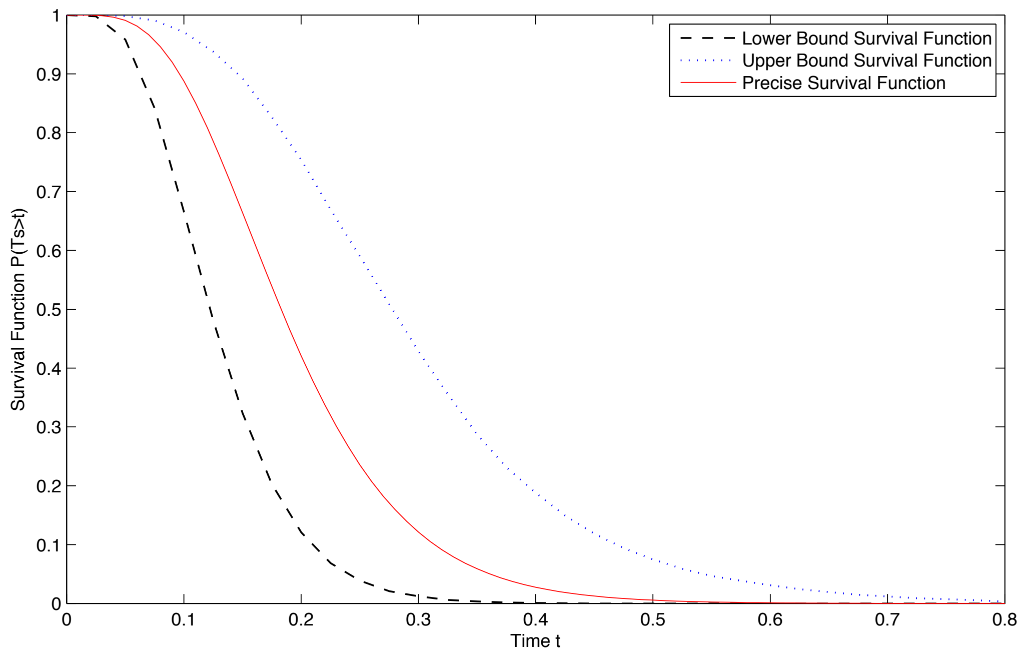

Figure 5: Time varying precise survival function alongside with lower and upper bounds of survival function of the network in Figure 4 (imprecise distribution parameters).